Analysis on the Effect of Geometric Deployment and Velocities of Sensors Based on TDOA/FDOA Measurement

ABSTRACT
In passive emitter location systems using the TDOA and FDOA measurements, the estimation performance varies with the geometric deployment and velocities of the sensors due to the high non-linearity of the TDOA/FDOA signal models, even when the estimation algorithm and the measurement error are not changed. To achieve a desired performance, finding appropriate deployment and velocities of the sensors is required. The problem is how to find them because there are too many cases of operating the emitter location systems. To cope with this problem, we derive an error ellipse to evaluate the estimation accuracy according to geometric deployment of the sensors based on the signal model in a 3-dimensional space (3-D) and various velocities. By using the derived error ellipse, analysis on the performance in the cases of several practical scenarios is carried out. The reliability of our evaluation method is confirmed by comparing to the performance evaluation based on the root-mean square error for the corresponding simulation results. Our analysis results on the various practical situations will be very helpful for operating the passive emitter location systems.

KEY WORDS
Emitter localization, TDOA, FDOA, Error ellipse

1. Introduction
Modern passive emitter location systems commonly use time difference of arrival (TDOA) and frequency difference of arrival (FDOA) measurements. TDOA and FDOA measurements yield a quadratic surface in a 3-dimensional (3-D) space. The emitter position is estimated from the intersection of three or more surfaces according to the measurements. The emitter location system has two major steps: TDOA/FDOA measurement and the estimation of emitter position. Firstly, TDOA and FDOA are estimated from the received radar signals radiated by the emitter at a number of sensors. Secondly the location of the emitters is estimated by using a specific algorithm based on TDOA and FDOA measurements. To achieve more precise measurement, the cross ambiguity function (CAF), which has been studied in active radar signal processing, was introduced [1], [2]. Recently, wideband processing using short-time CAF’s was proposed in [3]. Also, many approaches have been presented to estimate the emitter location by using a number of measurements [4]-[7]. The effect of position error of the receivers was considered in [7]. However, these previous researches focused on the enhancement of location accuracy without the consideration of geometric deployment of the sensors. To achieve a desired performance, finding appropriate deployment and velocities of the sensors is required because the shape and area of the intersection region is affected by the geometric position and velocities of sensors. There are too many scenarios with various deployment and velocities of the sensors. Since it is not desirable to evaluate the performances case by case for various scenarios, we derive an error ellipse which is used for evaluating the estimation performance in various practical scenarios. By using the derived error ellipse, we effectively analyze the effects of the geometric deployment and velocity of sensors on the estimation accuracy of emitter position without case-by-case simulations. The error ellipse is derived from the Cramer-Rao lower bound (CRLB) which is determined by the TDOA/FDOA signal models with the given positions and the velocities of the sensors.

In Section 2, the signal models of TDOA and FDOA are explained. Analysis on the simulation results according to the various scenarios is given in Section 3. Section 4 concludes this paper.

2. TDOA/FDOA signal models and CRLB
In this section, we apply the CRLB to TDOA/FDOA signal models when the sensors move with the uniform velocity and the emitter is fixed with the known altitude. Fig. 1 is the geometry of the sensors and emitter in 3-D space. To estimate unknown position of stationary emitter, \( \mathbf{u} = [x, y, z]^T \), \( M \) sensors are used whose position and
velocity are denoted by \( s_i = [x_i, y_i, z_i]^T \) and \( s_j = [x_j, y_j, z_j]^T \), \( i=1,2,3,...,M \), respectively. Let \( r_{ij} \) be the distance between emitter and \( i \)-th sensor, and is given by

\[
 r_{ij} = \| u - s_i \|. 
\]  (1)

Then TDOA between the sensor \( i \) and \( j \) is defined by

\[
 \tau_{ij} = \frac{r_{ij}}{c} + n_i = \frac{1}{c}(r_i - r_j) + n_i, 
\]  (2)

where \( c \) is the velocity of electromagnetic wave and \( n_i \) is the TDOA measurement error. From the time derivative of (2), FDOA is defined as follows:

\[
 v_{ij} = \frac{1}{c}\left( (u - s_i)^T \dot{s}_i - (u - s_j)^T \dot{s}_j \right) + n_i, 
\]  (3)

where \( n_i \) is the measurement error. As shown in (2) and (3), the TDOA measurement produces a hyperbola surface in the 3-D space, while the surface of the FDOA measurements is quite complicated due to the velocity vectors of the sensors.

To analyze the estimation performance according to the geometric deployment of the sensors, we use CRLB, which is the covariance of minimum variance unbiased estimator [9]. In other words, the CRLB represent best performance of estimation. We can confirm the optimum performance by comparing the CRLB corresponding to the various geometric deployments. The CRLB matrix is inverse matrix of the fisher information matrix, \( J \), and is given by

\[
 C_{\text{CRLB}} = J^{-1} = \left( H' C J H \right)^{-1}. 
\]  (4)

where \( C \) is the covariance matrix of the measurement error. And \( H \) is the Jacobian matrix, which is the partial derivative of the TDOA and FDOA measurements with respect to the \( x, y, \) and \( z \):

\[
 H = \frac{\partial \mathbf{f}}{\partial u} = [h_x, | h_y, | h_z]. 
\]  (5)

\( f \) consists of the TDOA and FDOA measurements:

\[
 f = [\tau_1, \tau_2, \tau_3, ..., \tau_N, \nu_1, \nu_2, ..., \nu_N]^T, 
\]  (6)

where the maximum value of \( N \) is \( M \) and the minimum value is 2 because we need at least 3 measurements to estimate unknown parameter.

In this paper, we consider the unknown emitter located on flat earth surfaces. Therefore, we don’t need to estimate parameter \( z \) of emitter so that the Jacobian matrix is reduced as (7) by removing influence of parameter \( z \)[10].

![Fig. 2 Relation between the eigenvalues and eigenvectors and the ellipse](image_url)

\[
 H_i = \frac{\partial \mathbf{f}}{\partial u} = [h_x, | h_y, | h_z]. 
\]  (7)

From (7), the modified CRLB is 2 by 2 matrix, and is given by

\[
 C_{\text{CRLB}} = J^{-1} = \left( H_i' C J H_i \right)^{-1} = k. 
\]  (8)

This yields error ellipse within which the estimates falls with specific probability \( P \) which satisfies \( k=-2\ln(1-P) \), \( 0 < P < 1 \) [10]. The Eigenvalues of CRLB, \( \lambda_1 \) and \( \lambda_2 \), is the length of the Ellipse’s major and minor axis and eigenvectors, \( q_1 \) and \( q_2 \), are direction as shown Fig. 2. Consequently, the circular error probability (CEP) is calculated by

\[
 \text{CEP} \approx (3/4)\sqrt{a^2 + b^2}, 
\]  (9)

where \( a \) and \( b \) are major axis and minor axis respectively [11].

3. Analysis on the localization performance by position and velocity of sensors

To analyze impact by position and velocity of sensors on estimation performance, we fix other variables except the distance between sensors and velocity of all sensors in subsection 3.1 and 3.2 respectively. Fixed parameters are represented in table 1. We investigate the theoretical performance and the practical algorithm performance under 4 scenarios according to the different locations of emitter as follows: 1) \([1000 \ 1000 \ 0]^T\), 2) \([500 \ 20000 \ 0]^T\), 3) \([20000 \ 20000 \ 0]^T\), 4) \([20000 \ 5000 \ 0]^T\). Under these scenarios, we use CEP which is explained in section 2 for theoretical analysis and iterative Taylor-series linearization method for algorithmic analysis. The covariance matrix of TDOA and FDOA measurements for the CRLB and algorithm is given by
Table 1 Parameters for the simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation of TDOA noise</td>
<td>10 ns</td>
</tr>
<tr>
<td>Standard deviation of FDOA noise</td>
<td>10 Hz</td>
</tr>
<tr>
<td>Probability $P$</td>
<td>0.5</td>
</tr>
<tr>
<td>Carrier Frequency of transmitted signal</td>
<td>10 GHz</td>
</tr>
<tr>
<td>Altitude of all sensors</td>
<td>5000 m</td>
</tr>
</tbody>
</table>

\[
C = \begin{bmatrix}
    n_{s_i}^2 & 0 & \\
    0 & n_{s_j}^2 & \\
    & 0 & n_{s_k}^2
\end{bmatrix},
\]

where it consists of the diagonal elements which are noise power and off-diagonal components which are zero under the assumption that noises are uncorrelated.

### 3.1 Distance between sensors

This subsection investigates theoretical and algorithmic performance of estimation as distance of each sensor varies in circumstances that sensors are deployed on a straight line. Fig. 3 shows theoretical localization performance by computing the CEP under the scenario that the number of sensors is three and position of sensors are $s_1=[-d~0~0]^T$, $s_2=[0~0~0]^T$ and $s_3=[d~0~0]^T$, where $d$ is distance of each sensor and varies from 500 m to 5000 m. The velocities of all sensors are fixed at 300 m/s to examine the effect of the position only.

Although the Case 4 is impractical in real situation because of the accuracy of the estimation, we can grasp the tendency of performance as the distance of each sensor varies just like other cases. When the distance between sensors is relatively short, CEP is the longest in all cases. It means the performance of the estimation is the worst. As the distance increases, CEP which is calculated in all cases decreases. Consequently, we found out that the performance of the localization is improved as the distance between sensors increases. To confirm the results of theoretical analysis, we apply iterative Taylor-series linearization method to the corresponding environments above. The weighting matrix of this method is the inverse matrix of the noise covariance matrix and the number of ensembles is 2000. Also, initial position of estimate as an input of the algorithm is set by 90% of the true location of the emitter. Fig. 4 shows that the RMSE of the estimates also decreases as the distance between sensors increases, *i.e.*, the performance of the estimation is enhanced as the distance between sensors increase. This result confirms the conclusion of the theoretical analysis.

### 3.2 Velocities of sensors

To examine the impact of the velocity on estimation performance, we carried out repetitive simulations similar to subsection 3.1. The number of sensors is three like above, but the positions of sensors are fixed at $s_1=[-2000~0~0]^T$, $s_2=[0~0~0]^T$, and $s_3=[2000~0~0]^T$. Only the velocities of all sensors are changed from 100 m/s to 500m/s. Fig. 5 shows CEP as a velocity of sensor varies. We found out that the faster the velocity of a sensor, the higher CEP from this figure. Therefore, the performance of the emitter localization becomes better as the velocity of sensor becomes faster theoretically. Iterative Taylor-series linearization method is also used to reconfirm of theoretical analysis in this subsection. The simulation circumstance, the weighting matrix, the initial value of the algorithm and the number of ensembles are same as subsection 3.1. As shown in Fig. 6, RMSE of estimates has same result like the theoretical analysis via CRLB. The accuracy of the localization is improved as the velocity of sensors increases.
4. Conclusion

To analyze the effects of geometric deployment and velocities of sensors on the estimation accuracy of emitter position, we derive an error ellipse based on the CRLB. The CRLB matrix is deduced from the TDOA/FDOA signal models and the Jacobian matrix in 3-D space. To be specific, we presented the analysis results for the cases of the four scenarios by using CEP and Taylor-series linearization method. These analyses showed that the longer the distance between sensors and the faster the velocities of all sensors, the better the accuracy of the estimation. The analysis results are expected to be very useful for planning to determine maneuver of sensors in passive emitter location systems.

References


