Analysis of Performance Penalties in Hierarchical DQPSK Modulation Systems

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Abstract—In hierarchical modulation (HM) based on DQPSK modulation, the performance of the receiver is influenced by the modulation noise and the demodulation method for base layer. To design an HM system, these performance penalty factors should be analyzed. This paper presents useful information for designing HM systems by doing mathematical and numerical analysis on their performance penalties.

I. INTRODUCTION

The hierarchical modulation (HM) is a method of signal processing technique to combine two separated streams into a single stream for transmission and the base layer signal and the additional layer signal are called high priority (HP) and low priority (LP) signal, respectively [1]. Since the HM has an advantage of increasing data payload without using additional bandwidth while securing backward compatibility, the HM has been widely used in many broadcasting services, such as DVB-T, Media-FLO and AT-DMB. This technique is very useful for brand-new receivers which support HM but legacy receivers have to withstand the reception performance penalty caused by the additional LP data. To design the system with HM, this penalty should be analyzed and considered. For the QPSK-QPSK HM method (HM I), which is one of mainly used combinations, and other HM methods such as DAPSK-QPSK-QPSK HM method (HM I), which is one of mainly covered. In this paper, to present useful information for designing HM systems by doing mathematical and numerical analysis on their performance penalties.

II. PENALTIES BY DIFFERENTIAL DEMODULATION

To analyze the penalty of the original system, we should calculate the difference between carrier-to-noise ratio (CNR) and modulation noise ratio (MNR). The penalty of HM I is presented in [1] and we can see that symbol energy $E_S$ is also equal from [1]. The CNR of HM II is given

$$\text{CNR}_{\text{II}} = E_s / N_0 = 2(1 + \lambda^2) s_1^2 / N_0. \quad (1)$$

Here, $2s_1$ represents the minimum distance between HP symbols as illustrated in Fig.1 (a). Notation $N_0$ means noise power and $\lambda$ is the hierarchical modulation index which is defined as $s_2 / s_1$, where $2s_2$ represents the distance between LP symbols within one quadrant. However, the MNR of HM II is different from that of HM I because the constellation of hierarchically modulated received signal is spread by the differential demodulation as illustrated in Fig. 1. (b). The demodulated HM II signal can be expressed as follows:

$$d_{\text{II}}(m) = (z_{j,k} + \lambda c_{j,k}) \cdot z_{j-1,k} + (z_{j,k} + \lambda c_{j,k}) \cdot (\lambda c_{j-1,k}^\ast), \quad (2)$$

where $z_{j,k}, c_{j,k} \in \{e^{j\pi/4}, e^{j3\pi/4}, e^{j5\pi/4}, e^{j7\pi/4}\}$ for even symbols and $z_{j,k}, c_{j,k} \in \{e^{j\pi/2}, e^{j3\pi/2}, e^{j5\pi/2}, e^{j7\pi/2}\}$ for odd symbols. Here, $z_{j,k}$ and $c_{j,k}$ denote HP and LP symbols for HM II, respectively. $I$ is a OFDM symbol index and $k$ is a subcarrier index. In (2), all terms except for the term of $z_{j,k}^\ast$ correspond to modulation noise whose power $N_{\text{SNR}}$ is calculated by

$$N_{\text{SNR}}^\text{II} = \frac{1}{M} \sum_{m=0}^{M-1} (d_{\text{II}}(m) - E[d_{\text{II}}(m)])^2 = 2(2 + \lambda^2)s_2^2. \quad (3)$$

Fig. 1. Constellations of various hierarchically demodulated signals: (a) The constellation of HM I at $\lambda = 0.2$, (b) The constellation of HM II after demodulation at $\lambda = 0.2$, (c) The constellation of HM III after demodulation at $\lambda = 0.2$, (d) The constellation of HM IV after demodulation at $\mu = 2$. [1],[4]-[6].

To design HM systems using DQPSK modulation, we analyzed penalties of HM II, III and IV. To verify the derived penalties and show the performance degradation by various differential modulation methods, we performed computer simulations.
where $d^*(m)$ means an element of the set which is consisted of all possible cases of $d^*_{1,k}$ and $M$ denotes the size of the set. Using (3), MNR of HM II is evaluated as follows:

$$
\text{MNR}_{\text{II}} = \frac{E^{\text{HP}}}{N_0} = \frac{1}{1 + \lambda^2 [1 + (2 + \lambda^2) \text{CNR}_{\text{II}}]}.
$$

where $E^{\text{HP}}$ is the energy of a DQPSK modulated symbol. The penalty of HM II, $P_{\text{II}}$ is the ratio between CNR and MNR and we can get it by rewriting (4) as follows

$$
P_{\text{II}} = \frac{\text{CNR}_{\text{II}}}{\text{MNR}_{\text{II}}} = 1 + \lambda^2 [1 + (2 + \lambda^2) \text{CNR}_{\text{II}}].
$$

Using similar ways, we can find the penalty of HM III and it is easily shown to be equal to that of HM II. Finally, to evaluate the penalty of HM IV, we firstly get the constellation of HM IV. In A-DPSK, HP part of the received symbol $r_{1,k}$ is the same as (3) and it is expressed as $r_{1,k} = z_{1,k}e^{j\psi_{1,k}}$ where LP part $c_{1,k}$ is $\{s_3, s_3 + s_3, s_3 + 2s_3, s_3 + 3s_3\}$. Here, $s_3$ denotes the minimum distance between the origin and the LP symbol and $s_4$ is the distance between LP symbols in a quadrant. For HM IV, the HM index $\mu$ is defined as $s_j/s_k$ [6]. To evaluate the CNR, we firstly get the mean and variance of the transmission symbol for HM IV. By calculating $r_{1,k}$, we can obtain all possible transmission symbols for $c_{1,k}$ and then we get the mean and variance using those symbols. Mean is $e^{j\theta_{1,k}}(s_3 + 1.5s_4)$ at $n=1, 3, 5, 7$ and variance is $2s_3^2 - 1.25s_3^2$. Using these results, the CNR is achieved as

$$
\text{CNR}_{\text{IV}} = \frac{E^{\text{HP}}}{N_0} = 2s_3^2 (1 + 1.25s_3^2)/N_0.
$$

The next step for evaluating $P_{\text{IV}}$ is to get $N_{MK}$ of HM IV signal. The demodulated symbol of HM IV is expressed by $d^*_{1,k} = r_{1,k} \cdot r_{1,k}^* = (z_{1,k} \cdot z_{1,k}^*) [c_{1,k}^N \cdot c_{1,k}^N]^T$. We can calculate the $N_{MK}$ and it is $2s_3^2 \cdot s_4^2 \cdot C/16$ where $C = 40\mu^2 + 120\lambda + 115$. Using these results, the MNR of HM IV is calculated by

$$
\text{MNR}_{\text{IV}} = \frac{E^{\text{HP}}}{N_0 + N_{MK}} = \frac{16 \cdot \text{CNR}_{\text{IV}}}{16 + s_3^2 (20 + \text{CNR}_{\text{IV}} \cdot s_4^2 \cdot C)}.
$$

Therefore, $P_{\text{IV}} = 1 + s_3^2 [1.25 + s_4^2 \cdot C \cdot \text{CNR}_{\text{IV}}]/16$. These penalties show the performance degradation by $\lambda$ but they are not the exact receiver performance. Since now modern broadcasting systems have adopted the FEC decoders, they may be changed according to the FEC decoding. The optimized method had been presented in [7] as $r_{1,k} \cdot r_{1,k}^*$ (DM I) but there is no guarantee that most receivers use DM I. To analyze the performance degradation by differential demodulation method, we also considered other methods: $|r_{1,k}| \exp(j \angle r_{1,k} \cdot r_{1,k}^*)$ (DM II), $r_{1,k} \cdot r_{1,k}^*$ (DM III), and $\exp(j \angle r_{1,k} \cdot r_{1,k}^*)$ (DM IV).

### III. Simulation Results

To verify the analyzed performance penalties and show the performance degradation by various demodulation methods, we performed computer simulations. In the computer simulation, the simulation is performed with and without HM II and used demodulation methods are DM I and DM II (the other results are omitted due to the page limit). The hierarchy parameter $\lambda$ is 0.2 and other parameters are presented in [4]. Simulation results are presented in table I. The penalty of HM II is approximately 1.45 dB at CNR = 5.1 dB but the real penalty by HM I is 1 dB. This difference is due to the coding gain. As shown in table I, the performance difference of DM I and DM II is increased as vehicle speed is changed whether additional data is loaded or not. This fact means that the performance of legacy receiver may be also changed by the selection of demodulation method.

### IV. Conclusions

This paper analyzed the causes of the performance penalties in hierarchical DQPSK modulation systems: modulation noise and HP demodulation methods. Since the result may be very usefully utilized for designing a hierarchically modulated system, it is expected to provide a guideline for the development of broadcasting systems with hierarchical modulation.

### References


### Table I

<table>
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<tr>
<th>Vehicle Speed</th>
<th>DM I (w/o HM)</th>
<th>DM II (w/o HM)</th>
<th>DM I (HM II)</th>
<th>DM II (HM II)</th>
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<tbody>
<tr>
<td>AWGN</td>
<td>5.1 dB</td>
<td>5.3 dB</td>
<td>6.1 dB</td>
<td>6.3 dB</td>
</tr>
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<td>5 km/h</td>
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<td>8.8 dB</td>
<td>8.8 dB</td>
<td>9.7 dB</td>
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<td>60 km/h</td>
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<td>9.3 dB</td>
<td>9.6 dB</td>
<td>10.2 dB</td>
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<td>120 km/h</td>
<td>9.3 dB</td>
<td>10.2 dB</td>
<td>10.6 dB</td>
<td>11.3 dB</td>
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<tr>
<td>200 km/h</td>
<td>9.7 dB</td>
<td>11.3 dB</td>
<td>11.1 dB</td>
<td>12.4 dB</td>
</tr>
<tr>
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<td>12.2 dB</td>
<td>14.8 dB</td>
<td>15.1 dB</td>
<td>17.2 dB</td>
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**SUMMARY OF BER PERFORMANCE OF THE LEGACY T-DMB RECEIVERS**