Maximum-likelihood angle estimator for multi-channel FM-radio-based passive coherent location

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Abstract: Frequency-modulation-based (FM-based) passive coherent location (PCL) systems can estimate the target location/velocity by exploiting a single FM channel. In order to improve the performance of PCL systems, multi-channel-single-transmitter setup-based PCL systems have been studied recently. However, it has not yet been considered for direction of arrival estimation based on the multi-channel configuration. Thus, the authors propose a maximum-likelihood angle estimator exploiting the multiple FM-radio channels and derive the Cramer-Rao bound for verifying the convergence of the proposed method in the sense of root-mean-square error (RMSE). In addition, they propose a target association algorithm for solving the ambiguity problem in selecting the steering vectors of each target. Computer simulations are included to show the estimation performance of the proposed method and to compare the RMSE of the single-channel case with that of the multi-channel case.

1 Introduction

Passive coherent location (PCL) is a passive radar technique that exploits the illuminators of opportunity such as FM (frequency modulated), digital video broadcasting-terrestrial, IEEE 802.11 standard-based Wi-Fi, and GSM (global system for mobile communications) transmitters [1–7]. The basic structure of the PCL system is a bistatic geometry consisting of a receiver, transmitter, and a target. As the receiver is located away from the transmitter, it receives the target echo signals and the direct-path signals at the surveillance and reference channels, respectively.

The location of the target may be estimated by using a multistatic structure with more than three bistatic geometries. From the transmitter–target–receiver distance called the bistatic range, the line of position of the target can be represented by an ellipse, the foci of which are the locations of the transmitter and the receiver. The target location is determined by the intersection of the multiple ellipses. Thus, it is important to detect the target echo signals emitted from the multiple transmitters [4, 8].

However, the detection of the target echo signals may not be guaranteed even when the targets are near the receiver [8]. To increase the detectability of the signals, direction-of-arrival (DOA) estimates can be used for finding the exact location of the target [8, 9]. As the DOA estimate of the target echo signal indicates the direction of the target from the receiver, the target location can be determined by finding the crossing point between the DOA and the ellipse even when only one target echo signal is detected.

There is much literature regarding DOA estimation for PCL systems. In [8], a direction-finding algorithm was developed both to decrease the influence of strong interference signals such as a direct-path signal and clutter echoes in the surveillance channel and to estimate the direction of the target echoes. However, the application of this DOA estimation algorithm is limited to the Adcock antenna array. In [10, 11], the multiple signal classification (MUSIC) algorithm [12] was adopted to estimate the DOAs of the target echo signals without any modifications for PCL systems. The signal subspace and the noise subspace are decomposed from the estimated covariance matrix of the received signal in the MUSIC algorithm. By using a property that the noise subspace is orthogonal to the signal subspace, the multiple DOAs can be estimated [12]. In the PCL systems, the information of the transmitted signal waveform can be obtained from the reference channel [9]. Since the estimation accuracy can be enhanced by exploiting the information of the signal waveform [12], the MUSIC algorithm may not be comparable to the DOA estimation algorithm using the signal waveform. Falcone et al. [6] proposed an experimental setup of a Wi-Fi-based multistatic passive radar including a DOA estimation block. However, they focused on the PCL system configuration and used a simple method involving the use of the phase difference between two antennas.

Among the various communication and broadcasting systems, one of the popular illuminators in the area of passive radars is an FM radio transmitter [4]. As the FM transmitter has a suitable transmitting power level for detecting targets as compared to other illuminators, the FM-based PCL can realise a wide detection coverage. However, the range resolution and range-estimation accuracy of the FM radio signal still require improvements. Owing to the narrowband property of the FM radio, it is difficult to obtain adequate range resolution and estimation accuracy of the parameters.

Therefore, a multi-channel structure of FM broadcasting has been considered. The conventional PCL system took into consideration only a single FM channel owing to the simplicity of its implementation [4]. However, as multiple carrier frequencies are allocated to an FM transmitter, as depicted in Fig. 1, there is no reason to limit the configuration to a single channel. Several studies on multi-channel FM-based PCL systems have been recently conducted in order to improve the performance of the parameter estimation and probability of detection of the target echo signal [13–17]. In addition, the range resolution of the FM signal can also be improved by exploiting the multi-channel FM structure [18, 19].

In this paper, we focus on the DOA estimation technique with a multi-channel configuration in the PCL system. There is little literature on the DOA estimation scheme for the multi-channel FM-based PCL. In [13, 20], the DOA estimation scheme was proposed in order to integrate the multi-frequency estimates of the target echo signals. Moreover, to integrate the multiple DOA estimates and to calculate the multi-frequency-based DOA estimate, a weighted sum of the multiple DOA estimates was proposed. However, the detailed derivation of the weighted sum was not revealed in [13, 20].

Multi-frequency-based DOA estimation techniques have been also proposed in other areas besides the PCL systems. In [21], the DOA estimation method for multi-band signals was proposed with the use of the hierarchical structure of the DOA estimates of the low- and high-frequency signals. Although this algorithm could improve the DOA estimation accuracy while preventing the
occurrence of the spatial aliasing problem, its advantage is limited to only the case wherein it generates spatial aliasing for a few carrier frequencies. In [22], the DOA estimation scheme based on the compressed sensing technique was proposed for multi-band signals. However, this DOA estimation algorithm operates in an environment in which the source signals are assumed to be unknown. In addition, its estimation accuracy for multi-band signals was not addressed in [22].

In order to improve the estimation accuracy of the direction of multiple-target echo signals by exploiting the multi-channel structure in the FM band, we propose a maximum-likelihood (ML) angle estimator for a multi-channel FM-based PCL system. Although the performance of the ML estimator generally approaches the Cramer-Rao bound (CRB), it has not been considered for a multi-channel-based PCL system. We also derive the CRB for the multi-channel FM structure in this paper. The derived CRB is used as a performance limit for the estimation accuracy of the multi-channel-based ML technique.

This paper is organised as follows. Section 2 presents the signal model and assumptions for the multi-channel-based FM signal. In Section 3, under the assumption that the direct-path signals are known, the ML angle estimator for a multi-channel FM signal is proposed and the multi-channel-based CRB is derived. The simulation results of the multi-channel ML are presented in Section 4, and the conclusions of this study are presented in Section 5.

2 Signal model

PCL geolocation that exploits the multiple FM channels used for the DOA estimation is presented in Fig. 2. The rth transmitter emits the multiple FM signals using a set of carrier frequencies, \( f_r = \{ f_r^{(1)}, \ldots, f_r^{(N_{ch})} \} \), where \( N_{ch} \) denotes the number of detected FM signals emitted from the rth transmitter. Since the multi-channel-multi-transmitter setup presented in Fig. 2 can be divided into several multi-channel-single-transmitter setups, without loss of generality, we consider only a multi-channel-single-transmitter setup as described in [17, 19]. Thus, the set of frequencies of the multiple FM signals emitted from a single transmitter can be represented as \( f = \{ f^{(1)}, \ldots, f^{(N_{ch})} \} \) by removing the subscript \( r \).

The number of detected target echo signals is assumed to be the same for all the channels, i.e. the number of detected target echo signals in the frequency band of the multiple channels is equal to \( N_{ch} \) for all the channels. As the target echo signals from the multi-channels have an equivalent target direction, all the target echoes may be used to estimate the target direction. Based on the PCL geolocation in Fig. 2, we derive a received-signal model of the multi-channel-single-transmitter case.

We also assume that strong interfering signals are removed by the interference suppression schemes such as ECA (extensive cancellation algorithm), FBLMS (fast block least mean square), NLMS (normalised least mean square), and RLS (recursive least squares) [23–26] and only the target echo signals are received from the surveillance channel. The receiver is configured with an antenna array of \( M \) antennas. When the incident angle of the received signal is denoted by \( \theta \), the time delay of the narrowband signal can be simply modelled using a steering vector, i.e.

\[
a(\theta) = [\exp(jk \cdot p_1), \ldots, \exp(jk \cdot p_M)]^T,
\]

where \( k = 2\pi f / \lambda \) denotes the wavenumber vector of the incident signal corresponding to the signal of wavelength \( \lambda \), \( p_m \) represents the position vector of the mth antenna, and \( e \) denotes the unit direction vector of propagation of the target echo signal.

The steering vector of the qth channel can be represented using the superscript \( (q) \) as \( a^{(q)}(\theta) = [a^{(q)}(\theta_1), \ldots, a^{(q)}(\theta_K)]^T \). When \( K \) targets exist, the array manifold matrix having column vectors comprising steering vectors is represented by

\[
A^{(q)}(\theta) = [a^{(q)}(\theta_1), \ldots, a^{(q)}(\theta_K)]^T,
\]

where \( \theta = [\theta_1, \ldots, \theta_K]^T \) denotes the incident angle vector. When \( K \) target echo signals in the qth channel are denoted by \( x^{(q)}(t_n) = [s^{(q)}(t_n), \ldots, s^{(q)}(t_n)]^T \), the received signal vector \( x^{(q)}(t_n) \) is represented by

\[
x^{(q)}(t_n) = A^{(q)}(\theta)x^{(q)}(t_n) + w^{(q)}(t_n), \quad n = 1, \ldots, N.
\]

where \( w^{(q)}(t_n) \) represents the white Gaussian random process in the qth channel. As \( s^{(q)}(t_n) \) also includes the amplitude components, \( \delta_k^{(q)}(t_n) \) can be rewritten as

\[
\delta_k^{(q)}(t_n) = \gamma_k^{(q)}s_k^{(q)}(t_n), \quad k = 1, \ldots, K.
\]

where \( \gamma_k^{(q)} \) is defined as a complex-valued amplitude of the FM signal reflected from the \( k \)th target in the qth channel, and \( s_k^{(q)}(t_n) \) denotes the target echo signal that has a normalised power, i.e. \( \sum_{k=1}^{K} |\gamma_k^{(q)}|^2 = 1 \). By substituting (4) into (3), (3) can be rewritten as

\[
x^{(q)}(t_n) = D^{(q)}(\theta)y^{(q)}(t_n) + w^{(q)}(t_n),
\]

where

\[
D^{(q)}(\theta) = A^{(q)}(\theta)\Gamma^{(q)},
\]

and \( \Gamma^{(q)} = \text{diag}\{\gamma_1^{(q)}, \ldots, \gamma_K^{(q)}\} \). The received signal model can be simplified as follows:

\[
x(t_n) = D(\theta)y(t_n) + w(t_n),
\]

where

\[
x(t_n) = [x^{(1)T}(t_n), \ldots, x^{(N_{ch})T}(t_n)]^T,
\]

\[
y(t_n) = [y^{(1)T}(t_n), \ldots, y^{(N_{ch})T}(t_n)]^T,
\]

\[
w(t_n) = [w^{(1)T}(t_n), \ldots, w^{(N_{ch})T}(t_n)]^T.
\]
where
\[ \Gamma = \text{blkdiag}(\Gamma^{01}, \ldots, \Gamma^{0N_\text{ch}}). \]

and
\[ D(\theta) = \text{blkdiag}(D^{00}(\theta), \ldots, D^{0N_\text{ch}}(\theta)). \quad (8) \]

The problem of interest in this paper is the estimation of \( \theta \) and \( \Gamma^{0q} \), \( q = 1, \ldots, N_\text{ch} \) under the assumption that the \( x^{0q}(t_n) \) is given. Since the signal vector \( y^{0q}(t_n) \) can be obtained from the reference channel and the signal power of the \( y^{0q}(t_n) \) is much stronger than other signal components, \( y^{0q}(t_n) \) is assumed to be known in this paper.

### 3 ML angle estimator and CRB for multi-channel-based PCL

In this section, the ML angle estimator for the multi-channel-based PCL is derived. In order to derive the multi-channel-based ML angle estimator, we assume that the number of detected target echo signals are the same for all the channels and the target echo signals for each channel are uncorrelated with each other.

#### 3.1 Derivation of the multi-channel-based ML

The log-likelihood function can be expressed as follows [27]:
\[ -\ln[W] = \text{tr}\left\{ W^{-1} \sum_{n=1}^{N} [x(t_n) - D(\theta)y(t_n)] \right\}, \]

where \( \text{tr}\{\cdot\} \) is defined as the sum of the elements of the main diagonal of a square matrix, and \( W \) represents the covariance matrix of \( w(t_n) \). The array manifold matrix \( \hat{D}(\theta) \) can be estimated by minimising the following function:
\[ J = \left| \frac{1}{N} \sum_{n=1}^{N} [x(t_n) - D(\theta)y(t_n)] [x(t_n) - D(\theta)y(t_n)]^H \right|. \quad (10) \]

Let
\[ \hat{R}_{xx} = \frac{1}{N} \sum_{n=1}^{N} x(t_n)x^H(t_n). \quad (11) \]
\[ \hat{R}_{xy} = \frac{1}{N} \sum_{n=1}^{N} x(t_n)y^H(t_n). \quad (12) \]

and
\[ \hat{R}_{yy} = \frac{1}{N} \sum_{n=1}^{N} y(t_n)y^H(t_n). \quad (13) \]

As the inter-channel covariance matrix of \( y^{0q}(t_n) \) and \( y^{0r}(t_n) \) for \( q \neq r \) is a zero-valued matrix, the covariance matrices in (11)–(13) can be represented by the block diagonal matrices of \( R_{yy}^{0q} \), \( \hat{R}_{xx}^{0q} \), and \( \hat{R}_{yy}^{0q} \), where
\[ \hat{R}_{xx}^{0q} = \frac{1}{N} \sum_{n=1}^{N} y^{0q}(t_n)x^{0q^H}(t_n). \quad (14) \]

\( \hat{R}_{yy}^{0q} \) and \( \hat{R}_{yy}^{0q} \) can be similarly defined as in (14).

Using (11)–(13), the function in (10) can be rewritten as
\[ J = \left[ \hat{R}_{xx} - D(\theta)\hat{R}_{xy} - \hat{R}_{yy}^H D(\theta)^H + D(\theta)\hat{R}_{yy}^H D(\theta)^H \right] \]
\[ + \hat{R}_{xx} - \hat{R}_{yy} - \hat{R}_{yy}^H \hat{R}_{xy}^H. \quad (15) \]

To minimise the cost function \( J \), the array manifold matrix \( \hat{D}(\theta) \) can be calculated using [27]
\[ \hat{D}(\theta) = \hat{R}_{yy}^H \hat{R}_{xx}. \quad (16) \]

From the estimate of \( \hat{D}(\theta) \) in (16), the estimate of the noise covariance matrix can be obtained by using
\[ \hat{W} = \frac{1}{N} \sum_{n=1}^{N} [x(t_n) - \hat{D}(\theta)y(t_n)][x(t_n) - \hat{D}(\theta)y(t_n)]^H \]
\[ = \hat{R}_{xx} - \hat{R}_{yy} \hat{R}_{yy}^H \hat{R}_{xx}. \quad (17) \]

As the cost function \( J \) in (15) is defined as a function of \( \theta \) and \( y^{0q} \) for all \( q \) and \( k \), the multi-dimensional search over all unknown parameters may be performed on (15). However, it is difficult to perform the multi-dimensional search on (15) owing to the involved computational complexity. To reduce the computational burden, a decoupling of each target echo signal can be applied to the multi-channel case as in [27, 28]. By decoupling each component of the targets, the likelihood functions can be separated from each other and the multi-dimensional search need not be performed.

According to Theorem 1 in [27], the cost function \( J \) is asymptotically equivalent to
\[ J \approx \text{tr}\left\{ \hat{R}_{xx}[D(\theta) - \hat{D}(\theta)] \hat{W}^{-1}[D(\theta) - \hat{D}(\theta)]^H \right\}. \quad (18) \]

To estimate the DOAs of \( K \) targets, the multi-dimensional search of (18) is still required. As the target echo signals are assumed to be uncorrelated, \( R_{yy} \) can be assumed to be a diagonal matrix.

The steering vectors that are associated with the \( k \)-th target in \( \hat{D}(\theta) \) can be exploited to estimate \( \theta_k \). The cost function corresponding to the \( k \)-th target may be written as
\[ J_k = \text{tr}\left\{ [F(\theta_k) - \hat{F}_k]^H \hat{W}^{-1} [F(\theta_k) - \hat{F}_k] \right\}, \quad (19) \]

where
\[ F(\theta) = \text{blkdiag}\{d^{00}(\theta), \ldots, d^{0N_\text{ch}}(\theta)\} \]
(20)

and
\[ \hat{F}_k = \text{blkdiag}\{d^{00}(\theta_k), \ldots, d^{0N_\text{ch}}(\theta_k)\}. \quad (21) \]

\( \hat{F}_k \) includes the estimated steering vectors of the \( k \)-th target for all the channels. For example, when \( K = 3 \) targets exist and \( N_\text{ch} = 2 \) channels are exploited, then \( \hat{F}_k \) is a block diagonal matrix of the steering vectors for \( k = 1 \) target, as shown in Fig. 3. It should be noted that (20) and (21) are defined as matrices with the steering vectors for all channels as the block diagonal components. From (19)–(21), the unknown parameters such as the amplitudes and the DOAs of the targets can be estimated by using
\[ \arg \min_{\Gamma, \theta} \text{tr}\left\{ \Gamma[Z(\theta_k) - \hat{F}_k]^H \hat{W}^{-1} [\Gamma[Z(\theta_k) - \hat{F}_k] \right\}. \quad (22) \]
where $\Gamma_k = \text{diag} \{ \gamma_k^{(1)}, \ldots, \gamma_k^{(N_{\text{ch}})} \}$ and $Z(\theta_k) = \Gamma_k F(\theta_k)$. $Z(\theta_k)$ represents a block diagonal matrix of the steering vectors $\sigma_{\theta}(\theta_k)$ for all the FM channels.

All the amplitude parameters in $\Gamma_k$ are inherently uncorrelated with each other. From the partial differentiation of (22) with respect to $\gamma_k^{(q)}$, the amplitude parameters of the target echo signals can be obtained by using

$$\gamma_k^{(q)} = \frac{\sigma_{\theta, H}(\theta_k) W_k^{\omega q} \gamma_k^{(q)}}{\sigma_{\theta, H}(\theta_k) W_k^{\omega q}}.$$

The estimate of the DOAs $\hat{\theta}_k$ can be derived by substituting $Z(\theta)$ for (23) and partially differentiating (23) with respect to $\theta_k$. We then obtain

$$\hat{\theta}_k = \arg \max_{\theta_k} \left[ \sum_{q=1}^{N_{\text{ch}}} \sigma_{\theta} W_k^H \gamma_k^{(q)} \right].$$

### 3.2 Cramer-Rao bound

The main difference between a single-channel and a multi-channel configuration is that the inter-channel correlation is considered using Fisher information. A log-likelihood function of (5) is represented as follows:

$$\ln L = -NMN_{\text{ch}} \ln \det W - \sum_{q=1}^{N_{\text{ch}}} \sum_{n=1}^{N_q} e_{\theta}^{(q)}(t_n) W_{\omega q}^{-1} e_{\theta}^{(q)}(t_n),$$

where

$$e_{\theta}^{(q)}(t_n) = x_{\theta}^{(q)}(t_n) - D_{\theta}^{(q)}(\theta) y_{\theta}^{(q)}(t_n),$$

and $W = \text{blkdiag}(W^{(1)}, \ldots, W^{(N_{\text{ch}})})$. Let $\sigma_{\theta} = [\sigma_{\theta}^{(1)}, \ldots, \sigma_{\theta}^{(N_{\text{ch}})}]^T$ and $\sigma = [\sigma_{\theta}^T, \gamma_{\theta}^T, \Theta_{\theta}^T]^T$ denote the total elements of the noise covariance matrix, and the unknown parameter vector can be written as

$$\psi = [\sigma^T, \gamma^T, \Theta^T]^T.$$

Then, the Fisher information matrix $I(\psi)$ can be derived by using

$$I(\psi) = E \left[ \frac{\partial \ln L}{\partial \psi} \frac{\partial \ln L}{\partial \psi^T} \right].$$

### 3.3 Ambiguity problem in the multi-channel-based ML

In this subsection, we consider the association problem of the steering vectors in $D(\theta)$. To estimate $\theta_k$ by using the multi-channel-based ML technique, the $k$th steering vectors in each $D_{\theta}^{(q)}(\theta)$ are required to be extracted. However, there is an ambiguity problem in the selection of the steering vectors caused by the $k$th target. Thus, we are required to find out whether the steering vectors are estimated in the same target.

### 3.3.1 Problem formulation: If we assume that the known waveform $y(t_n)$ can be permuted using $P$, the permuted known waveform $\tilde{y}(t_n)$ may then be represented by

$$\tilde{y}(t_n) = Py(t_n),$$

where $P = \text{blkdiag}(P^{(1)}, \ldots, P^{(N_{\text{ch}})})$ denotes the block diagonal matrix of the permutation matrices. The multiplication of the permutation matrix in (35) indicates that the association to the targets is unclear. The permutation matrix $P$ may be assumed to be unknown.

We first consider the effect of $P$ in the estimation of $\hat{D}(\theta), \hat{W}$, and $\hat{\Gamma}$. Using (35), the estimate of the covariance matrices in (12) and (13) may be rewritten as

$$\text{CRB}_{\text{multi-channel}}(\hat{\theta}_k) = \frac{\text{CRB}_{\text{single-channel}}(\hat{\theta}_1)}{N_{\text{ch}}},$$

Therefore, under the condition that the estimation accuracy of the ML technique approaches the CRB derived in (33), the variance of $\hat{\theta}_k$ for the single-channel case is approximately equal to $N_{\text{ch}}$ times the variance of $\hat{\theta}_k$ for the multi-channel case.
In order to associate the steering vectors of the same target, the inter-channel correlation between each steering vector may be simply calculated using

\[ U^{q} = \tilde{D} (\theta) \tilde{D}^H (\theta), \]  

(43)

where \( q = 1, \ldots, N_{ch} \). As \( U^{q} \) represents the approximation of the permutation matrix \( P^{q} \), the estimate of the permutation matrix \( \hat{P}_{q}^{(n)} \) may be obtained by exploiting the indices that represent the maximum values in the column vectors of \( U^{q} \). When we define \( \hat{u}_{q}^{(n)} \) and \( \hat{t}_{q}^{(n)} \) as the \( q \)th column vector of \( U^{q} \) and \( \tilde{D}^{q} \), respectively, then \( \hat{P}^{(n)}_{q} \) may be calculated using

\[ \hat{P}^{(n)}_{q} = \hat{t}_{q}^{(n)} \hat{u}_{q}^{(n)} \]  

(44)

where \( T(\cdot) \) represents a transformation in which the maximum value in \( u_q^{(n)} \) is set to 1 and the remaining components are changed to 0. Then \( \hat{P}^{(n)}_{q} \) can be obtained by using

\[ \hat{P} = \text{blkdiag} \left[ \hat{P}_{1}^{(n)}, \ldots, \hat{P}_{K}^{(n)} \right]. \]  

(45)

Thus, \( \hat{P} \) can be obtained by using

\[ \hat{\theta} = \text{blkdag} \left[ \hat{\theta}_{1}, \ldots, \hat{\theta}_{K} \right]. \]  

(46)

Using (35) and (46), the non-permuted source signal \( \tilde{y}_{(n)} \) may finally be estimated by using

\[ \tilde{y}_{(n)} = \hat{P} \tilde{y}(\hat{\theta}). \]  

(47)

### 4 Simulation results

We present the root mean square error (RMSE) of the DOAs of the target according to the signal-to-noise ratio (SNR). When we denote the \( \hat{\theta}_{i} \) \((i = 1, \ldots, L)\) as the estimate of the DOAs at the \( i \)th trial, then the RMSE can be obtained as

\[ \text{RMSE} = \sqrt{\frac{1}{L} \sum_{i=1}^{L} (\hat{\theta} - \hat{\theta}).} \]  

(48)

where \( L \) represents the number of Monte Carlo simulations.

We consider a uniform linear array with \( M = 8 \) sensors. In addition, we assume that the inter-element spacing of the uniform linear array satisfies \( d = \lambda_{\text{avg}} / 2 \) where \( \lambda_{\text{avg}} \) denotes the minimum value of the wavelength of the multiple channels of interest. The received signals of the multiple channels are sampled with \( f_s = 300 \text{kHz} \), and the signals are observed for 1 ms. The observation time of 1 ms may not be sufficient to achieve the reliable performance of the target localisation. Nevertheless, the observation time of 1 ms was used to show the convergence of the RMSE to the CRB in the SNR values lying in the range from \(-35 \) to \(-5 \text{ dB} \) because the short observation time results in distinct convergence performance. Since the observation time is strongly related to the memory storage size and processing time, its length is meaningful for performance comparison. \( L = 500 \) Monte Carlo simulations are performed in each simulation.

#### 4.1 Root-mean-square error

First, we assume that \( N_{ch} = 4 \) channels of the target echo signals are received, and each channel has carrier frequencies of 95.9, 98.7, 106.1, and 107.7 MHz. We also consider that \( K = 4 \) targets exist, and the DOA of the target is \( \theta = 0.0873 \text{ rad} \). Fig. 4 shows the RMSE of the single-channel and the multi-channel MLs according to the SNR. As shown in Fig. 4, the RMSE of the multi-channel ML technique has a lower value than that of the single-
channel ML. Considering that the RMSE approaches the CRB, the difference in the RMSE between the \( q \)th single-channel and multi-channel can be calculated using (33). When the wavelengths for the multi-channel signals are approximately the same, the difference in the RMSE can be calculated using \( 10\log_{10}(N_{\alpha}) \). For example, in Fig. 4, the difference between the RMSE of the single-channel and that of the multi-channel ML is calculated to be \(-6.02 \text{ dB} \) as shown in (34). In addition, the RMSE of the proposed method converges to the CRB at lower SNR value than that of the single-channel cases. This means that the proposed method has the better convergence performance than the single-channel-based DOA estimation technique.

Second, we increase the number of channels as \( N_{ch} = 6 \) and the RMSE is shown in Fig. 5. The carrier frequencies are \( 89.1, 91.9, 95.9, 98.7, 106.1, \) and \( 107.7 \text{ MHz} \). As six channels are exploited, the difference between the single-channel and multi-channel MLs is \(-7.78 \text{ dB} \). As in case of Fig. 4, the RMSE of the proposed method converges to the CRB at much lower SNR value than that of the single-channel cases.

Third, the derived RMSE according to the number of snapshots is shown in Fig. 6. The dotted lines represent the CRBs. \( N_{ch} = 5 \) channels are exploited and the set of carrier frequencies is the same as the set of carrier frequencies used in Fig. 5, except for the frequency of \( 98.7 \text{ MHz} \), which is not included. For each channel, the SNR values are set as \(-26, -24, -22, -20, \) and \(-18 \text{ dB} \). As shown in Fig. 6, the RMSE of the multi-channel-based ML approaches the CRB as the number of snapshots increases. As the multi-channel-based ML exploits all target echo signals of various

\[
\text{SNR values, the RMSE of the proposed method has lower values than that of the single-channel-based ML technique.}
\]

### 4.2 Association problem

We consider the association problem in the multi-channel ML technique. \( N_{ch} = 4 \) channels are used, and \( K = 2 \) targets exist. The observation time is set to 2 ms. The carrier frequencies are equivalent to the frequencies in Fig. 4. The performance of the multi-channel-based DOA estimation algorithm presented in [20] is compared to the proposed method. In [20], the phase difference corresponding to each FM channel is derived. This algorithm is limited to antenna configuration only with two antennas. From the phase difference of the multi-channels and the weighted sum of the phase difference, the DOA estimation can be performed.

\[
\Phi^{\circ} = \Phi = \frac{\hat{d}^{(1)}(\theta_1) + \hat{d}^{(2)}(\theta_2)}{2},
\]

and

\[
\Phi^2 = \Phi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.
\]

The permutation matrices in (49) and (50) represent that \( \hat{d}^{(1)}(\theta_1) \) and \( \hat{d}^{(2)}(\theta_2) \) are associated with \( \hat{d}^{(1)}(\theta_1) \) and \( \hat{d}^{(2)}(\theta_2) \) in the multi-channel ML. The RMSE according to the SNR is given in Fig. 7. The RMSE of the multi-channel ML technique, which is not applied to the target association algorithm, is diverged. In contrast, the RMSE of the ML technique using the target association algorithm approaches the CRB. The target association problem can also be observed in Fig. 8. The SNR values of two target echo signals are set as \(-20 \text{ dB} \). As shown in Fig. 8a, each likelihood function of the targets has local maximum values at both \( \theta = -20^\circ \) and \( \theta = 25^\circ \). On applying the target association technique, each likelihood function indicates the maximum value at the DOA of the target as shown in Fig. 8b.

### 4.3 Performance comparison

The performance of the multi-channel-based DOA estimation algorithm presented in [20] is compared to the proposed method. In [20], the phase difference corresponding to each FM channel between the adjacent two antennas was derived. This algorithm is limited to antenna configuration only with two antennas. From the phase difference of the multi-channels and the weighted sum of the phase difference, the DOA estimation can be performed.

The phase difference \( \Phi^{\circ} \) at \( q \)th channel between two antennas can be derived by

\[
\Phi^{\circ} = \arg\left\{ \left| \phi_q^{(c)}(r, \nu) \right| \right\}, \quad q = 1, \ldots, N_{ch}.
\]

where \( \phi_q^{(c)}(r, \nu) \) denotes the bistatic range-velocity cross-correlation functions obtained by \( q \)th channel and \( m \)th antenna at the range \( r \)

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and the velocity $\nu$. To integrate the multi-channel-based phase difference $\Phi^q$, the weighted sum of the phase difference corresponding to the wavelength can be written as

$$\Phi^\alpha = \frac{1}{N_{ch}} \sum_{q=1}^{N_{ch}} \alpha(q) \lambda(q) \Phi^q,$$

where $\sum_{q=1}^{N_{ch}} \alpha(q) = 1$ and the weight vector $\alpha = [\alpha^1, \ldots, \alpha^{N_{ch}}]^T$ can be obtained by various criteria. In [20], the estimated SNR at each channel is proposed to be used as the weight vector $\alpha$. The multi-channel-based DOA estimation can be estimated by

$$\hat{\theta} = \sin^{-1} \left( \frac{\Phi^\alpha}{2\pi d} \right).$$

To calculate the estimation error, we consider an array with $M = 2$ antennas and the number of channels with $N_{ch} = 5$. The SNR values of each FM channel are set as $-30$, $-27$, $-24$, $-21$, and $-18$ dB and the carrier frequencies are the same as the frequencies in Fig. 6. To derive the estimation results proposed in [20], the following weight vectors $\alpha_1$, $\alpha_2$, and $\alpha_3$ are used:

$$\alpha_1 = [1, 1, 1, 1, 1]^T. \quad (54)$$

where $\alpha_1$ denotes the uniform weighting, $\alpha_2$ is weighted by the amplitude of the target echo signal and $\alpha_3$ is weighted by the signal power of the target echo signal.

Fig. 9 shows the estimation performance of the proposed method and the estimation method in [20]. The estimation performance of the proposed method outperforms the weighted-sum methods of (54)–(56). Although the number of snapshots increases, the RMSEs of the weighted-sum methods cannot approach the multi-channel-based CRB.

5 Conclusions

We derived the multi-channel-based ML angle estimator in the application of an FM-radio-based PCL system. By exploiting the property of the multi-channel-single-transmitter setup, we showed that the estimation accuracy can be improved through the use of multi-channel configuration. Furthermore, the target association problem that may be caused by the ambiguity in the target echo signals was also taken into consideration. To solve this association problem, we proposed an algorithm for estimating the permutation matrix, which resolved the issue of the ambiguity of the target echo signal. We also determined the relationship between the single-channel- and multi-channel-based CRB.

From the simulations, we found that the multi-channel-based ML angle estimator has a lower RMSE than that of the single-channel-based ML technique. We also found that the RMSE of the multi-channel signal asymptotically approaches the CRB.

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7 References

In this appendix, the CRB of the DOA estimation for the multi-channel-based PCL is derived. The first-order derivative of the log-likelihood function in (25) according to the unknown parameter $\psi$ is given by

$$\frac{\partial \ln L}{\partial \psi} = 2 \Re \sum_{q=1}^{N_q} \sum_{l=1}^{N} \left[ S^H(q^H(t_l) \hat{C}_{\psi}^H(\theta) \hat{C}_{\psi}^\dagger(\theta) S^\dagger(q^H(t_l)) \right].$$

In (64)–(66), the operator $\otimes$ represents the element-wise multiplication of two matrices. From (63)–(66), the Fisher information matrix in (28) can be rewritten as shown in (62).

For the derivation of the CRB of $\theta$, Remark 4 in [28], which represents one method for deriving the inverse matrix, can be used. Remark 4 [28] may be used for the non-singular complex matrix $X$ as
\[
\begin{bmatrix}
\text{Re}(X) & -\text{Im}(X) \\
\text{Im}(X) & \text{Re}(X)
\end{bmatrix}^{-1}
= 
\begin{bmatrix}
\text{Re}(X^*) & -\text{Im}(X^*) \\
\text{Im}(X^*) & \text{Re}(X^*)
\end{bmatrix}.
\] (67)

Finally, using (63)-(67), the CRB of \( \theta \) can be obtained as

\[
\text{CRB}^{-1}(\theta) = 2\Re \sum_{q=1}^{N} \left[ \mathbf{G}^{(q)} - \mathbf{G}^{(q)H} \mathbf{G}^{-1} \mathbf{G}^{(q)H} \right]
\] (68)

Assuming that \( \mathbf{R}_{ss}^{(q)} \) approaches \( \mathbf{R}_{ss} \) asymptotically, then the CRB of \( \theta_k \) is simplified as

\[
\text{CRB}^{-1}(\theta_k) = 2N \sum_{q=1}^{N} \left[ \mathbf{R}_{ss}^{(q)} \right] \mathbf{P}^{-1}_{a_k} \mathbf{d}_k \mathbf{d}_k^H.
\] (69)

where

\[
\mathbf{P}^{-1}_{a_k} = I - \left[ \mathbf{a}_{k} \mathbf{a}^H \right]^{-1} \mathbf{a}_{k} \mathbf{a}^H.
\] (70)

When the uniform linear array with \( M \) antennas is considered, then (69) can be rewritten as

\[
\text{CRB}(\theta_k) = \frac{6\sigma^2}{N(M^2 - 1) \cos^2(\theta_k) \sum_{q=1}^{N} \left[ \mathbf{R}_{ss}^{(q)} \right] \left( 2\pi d/\lambda^{(q)} \right)^2}.
\] (71)

where \( d \) denotes the distance between the adjacent antennas, \( \lambda^{(q)} \) represents the wavelength in the \( q \)th channel, and \( \mathbf{W} = \sigma \mathbf{I}_{MN_{ch}} \) is assumed in (71).