Two-step estimator for moving-emitter geolocation using time difference of arrival/frequency-difference of arrival measurements

Yong-Hee Kim, Dong-Gyu Kim, Hyoung-Nam Kim

Department of Electronics Engineering, Pusan National University, Busan 609-735, Korea
E-mail: hnkim@pusan.ac.kr

Abstract: This study presents a two-step least-squares approach for estimating the position and velocity of a moving emitter using time difference of arrival (TDOA) and frequency-difference of arrival (FDOA) measurements in a two-dimensional scenario. The proposed method first estimates the emitter position based on the asymptotes of TDOA hyperbolae and then linearises the non-linear equations of FDOAs using the estimate of the emitter position obtained in the first stage. Although the proposed method has lower performance than other methods in terms of estimation accuracy, it provides a closed-form solution without geometric constraints of sensor placement and an initial guess of emitter parameters. Since many methods derive the estimates of position and velocity with iterative numerical techniques, the proposed method can be a good alternative to set the initial guess under conditions of low measurement error. Simulation results are included to validate the algorithm, and its performance is compared with the Cramer–Rao lower bound.

1 Introduction

Passive emitter location has been of considerable interest in many fields, including radar, sonar and electronic warfare (EW). Especially in the area of electronic support (ES), estimating the position and the velocity of a moving emitter is critically important for evaluating the immediacy of moving threats on the ground or in the air. The ES system intercepts the electromagnetic signals radiated from an unknown emitter and analyses the intercepted signals to identify the type of an unknown emitter and estimate its position and/or velocity [1, 2]. Recently, demand for more accurate moving-emitter location has grown for establishing appropriate tactical strategies for EW activities. Although various physical quantities can be utilised for estimating the unknown position of an emitter such as range, angle and Doppler shift [3–5], many emitter location systems were based on the range measurements which are received signal strength, time of arrival and time difference of arrival (TDOA) [6, 7]. In particular, TDOA has been extensively studied to improve estimation accuracy with a low computational complexity for a stationary emitter, thereby iterative methods and direct solutions with some assumptions were presented [8–11]. When there is relative motion between the sensors and the emitter, frequency-difference of arrival (FDOA) can be used to estimate the velocity of a moving emitter as well as the emitter position. Hence, TDOA and FDOA measurements have been jointly utilised to simultaneously estimate emitter position and velocity [12–17].

The main issues in the TDOA/FDOA-based emitter location of a moving emitter are how to solve a set of non-linear TDOA/FDOA equations when the unknown position and velocity parameters are severely coupled in an FDOA signal model. To solve this problem, Ho and Xu [11] proposed a two-step weighted least-squares (WLSs) method called the quadratic correction least-squares (QCLSs) method. The QCLS method generates a pseudo-linear signal model by defining additional nuisance variables. The estimates of emitter position and velocity are obtained by solving the two-stage WLS algorithm. Many studies have followed this QCLS method to improve the estimation accuracy with lower computational complexity [12–16]. The non-linear nuisance parameters in the QCLS method were eliminated by employing an orthogonal projection matrix, resulting in greater computational simplicity [12]. A semi-definite relaxation method was applied to solve the problem of TDOA/FDOA-based moving-emitter location and enhance the estimation accuracy [13]. However, these QCLS-based methods require constrained conditions with the sensors neither lying in a plane nor on a straight line and all sensor pairs sharing the common reference sensor.

Several studies have tried to improve the accuracy by exploiting a constrained least-squares approach based on the technique of Lagrange multipliers instead of the two-stage WLS approach in the original QCLS method. The solution minimising the constrained least squares (LS) function was iteratively calculated using Newton’s method [14–16]. This method requires a proper initial guess to guarantee convergence to the global optimal solution, even though it is based on the pseudo-linear model of the QCLS method, which provides a closed-form solution for estimating the emitter position and velocity. There have also been non-QCLS-based iterative approaches that have tried to estimate emitter position and velocity or to track a moving emitter [17, 18]. The Taylor-series method finds the estimate of emitter position iteratively based on Taylor-series expansion [18]. An advanced extended Kalman filter method using TDOA and FDOA measurements from two receivers was presented [17]. Although these methods also require a proper initial guess to yield accurate estimates without the divergence problem, they can be used without the constrained conditions of sensor placements.

Both the QCLS-based methods [11–16] and the non-QCLS-based iterative methods [17, 18] suffer from limitations regarding the geometric constraints and initial guess, respectively. Therefore robust estimation methods without such limitations should be considered to develop a TDOA/FDOA-only emitter location system without a priori information. We present a two-step estimator based on TDOA and FDOA measurements. In the first stage, the position of a moving emitter is derived based on the asymptotes of TDOA hyperbolae, and then the angles of arrival (AOAs) are calculated from the position estimate. Using the estimated AOA, a set of FDOA equations can be linearised with respect to the emitter velocity. Although the method shows a relatively low estimation accuracy compared with other methods, its performance reaches the Cramer–Rao lower bound (CRLB) when the TDOA/FDOA measurement errors are small. In addition,
the method does not have any constraints of sensor placement or an initial guess, making it an appropriate alternative to provide a proper initial guess for iterative approaches only if TDOA and FDOA can be obtained with small measurement errors.

The proposed method is implemented in a two-dimensional (2D) scenario to generate a closed-form solution without geometric constraints. Several studies examined a 2D scenario [19–21], which is useful for many areas such as sensor networks and ground-based moving-emitter location. Particularly in military applications, when aircraft fly at very low altitude to detect an enemy’s radio transmitter, the approximated 2D problem can be sufficiently valid [22].

The rest of this paper is organised as follows. Section 2 provides a brief overview and problem description of TDOA/FDOA-based emitter location. The proposed two-step method for estimating the position and velocity of a moving emitter is presented in Section 3. In Section 5, simulation results are included to evaluate the performance of the proposed method. Finally, Section 4 concludes this paper.

2 Signal models for TDOA/FDOA measurements

A 2D scenario is considered to estimate the position \( \mathbf{x}_e = [x_e, y_e]^T \) and velocity \( \dot{\mathbf{x}}_e = [\dot{x}_e, \dot{y}_e]^T \) of a moving emitter using TDOA and FDOA measurements. For \( N \) spatially separated sensors, their positions \( \mathbf{s}_i = [x_i, y_i]^T \) and velocities \( \dot{\mathbf{s}}_i = [\dot{x}_i, \dot{y}_i]^T \) (\( i = 1, 2, \ldots, N \)) are assumed to be exactly known. \( \dagger \) denotes the transpose of a matrix. If \( M \) independent sensor pairs are combined, a total of 2M measurements are available, because each sensor pair produces a pair of TDOA and FDOA. The TDOA and FDOA measurements are determined by relative sensor-emitter geometry/velocity, and they are functionally related to the range difference and the range-rate difference, respectively [23].

The distance between the emitter and the sensor \( i \) is

\[
\mathbf{r}_i = |\mathbf{r}_i| = |\mathbf{x}_e - \mathbf{s}_i| = \sqrt{(x_e - s_{ix})^2 + (y_e - s_{iy})^2}, \quad i = 1, 2, \ldots, N
\]

where \( \mathbf{r}_i \) denotes the radius vector between the emitter and the sensor \( i \). The TDOA \( \tau_i \) between the sensor \( k \) and the reference sensor \( 1 \) is caused by the range difference, and they are related as follows

\[
\tau_k = \frac{1}{c} (r_k - r_1) = \frac{1}{c} (|x_e - s_{kx}| - |x_e - s_{s1x}|), \quad k = 2, \ldots, N
\]

where \( c \) is the speed of light and \( s_k \) is the position of a reference sensor. As shown in (2), the TDOA depends on only the positions of the emitter and sensors, so it can be utilised to estimate only the emitter position, not the velocity. When either the sensors or emitter move, a relative motion causes variation in the range between the sensor and the emitter, which is referred to as a range-rate. The range-rate is defined as the time derivative of (1) and is given by

\[
\dot{r}_i = \frac{(\mathbf{x}_e - \dot{s}_{ix})(\mathbf{x}_e - \dot{s}_{iy})}{r_i} = (\mathbf{x}_e - \dot{s}_i)^T \mathbf{i}_i \quad (3)
\]

where \( \mathbf{i}_i = (\mathbf{x}_e - \mathbf{s}_i)/r_i \) is a unit vector of the radius vector \( \mathbf{r}_i \) pointing from the sensor \( i \) to the emitter. The range-rate represents the scalar component of \( (\mathbf{x}_e - \dot{s}_i) \) in the direction of \( \mathbf{i}_i \), as depicted in Fig. 1. Since the Doppler frequency in a sensor is proportional to its range-rate in (3), the received frequency in the sensor \( i \) is obtained by

\[
f_i = f_0 + f_d = f_0 \left(1 - \frac{1}{c}(\mathbf{x}_e - \dot{s}_i)^T \mathbf{i}_i \right), \quad i = 1, 2, \ldots, N
\]

where \( f_0 \) and \( f_d \) denote the carrier frequency and the Doppler frequency, respectively. Then, the FDOA and the range-rate difference between the sensor \( i \) and the reference sensor \( 1 \) are related as follows

\[
f_k = \frac{f_0}{c} (r_k - r_1) = \frac{f_0}{c} \left(\frac{(x_k - \dot{s}_{kx})(y_k - \dot{s}_{ky})}{r_k} - \frac{(x_1 - \dot{s}_{s1x})(y_1 - \dot{s}_{s1y})}{r_1}\right), \quad k = 2, \ldots, N
\]

From (5), it is found that FDOA can be used for estimating the velocity vector and position of a moving emitter.

Since the signal models for TDOA/FDOA are assumed to be deterministic in the case of passively locating an electromagnetic emitter [23], the observed TDOA/FDOA measurements \( u^o \) corrupted by the additive noise are represented by

\[
u^o = u + w = \begin{bmatrix} \tau \\ f \end{bmatrix} + \begin{bmatrix} \Delta\tau \\ \Delta f \end{bmatrix}
\]

where \( u = [\tau^T, f^T]^T \) and \( w = [\Delta\tau^T, \Delta f^T]^T \) denote the true values of TDOA and FDOA and their measurement errors, respectively. The error vector \( w \) consists of \( \Delta\tau = [\Delta\tau_1, \Delta\tau_2, \ldots, \Delta\tau_M]^T \) and \( \Delta f = [\Delta f_1, \Delta f_2, \ldots, \Delta f_N]^T \), and it is assumed to be zero mean with a covariance matrix of [11]

\[
C_w = E[ww^T] = E[(\Delta\tau^T \Delta f^T)\Sigma(\Delta\tau^T \Delta f^T)]
\]

3 Two-step estimation for moving emitter geolocation

Estimating the position and the velocity of a moving emitter using the TDOA/FDOA measurements has been a challenging task because of the high non-linearity in the TDOA/FDOA signal models and the non-separability of the position and velocity parameters in FDOA equations. The most straightforward method is to linearise the TDOA and FDOA equations through Taylor-series expansion [18], which requires a proper initial guess close to the true position/velocity of a moving emitter. An alternative closed-form solution was presented [11] in which the TDOA and FDOA equations are transformed to a set of pseudo-linear equations by introducing nuisance parameters. However, the closed-form solution uses a geometrical constraint with the sensors neither lying in a plane nor in a straight line. Many approaches using the constrained least-squares resort to a numerical iterative algorithm, even though
they are based on a closed-form solution. Therefore we present a closed-form solution for estimating the emitter position and velocity without constraints of sensor deployment or the need for a proper initial guess in a 2D scenario.

The proposed method consists of two steps in which the position and the velocity of a moving emitter are estimated sequentially. In the first step, the closed-form asymptote intersection estimator replaces the problem of intersecting the TDOA hyperbolas and produces approximate estimates close to the true solution with the emitter at long range [19, 24]. Since TDOA depends only on the position of the emitter and sensors, as shown in (2), the emitter position can be derived without considering the velocity of the emitter and sensors. TDOA hyperbola can be approximated by their asymptotes in the far-field area, and this approximation is valid when the emitter is located at long range from the sensor pair, as shown in Fig. 2. The gradient angle of an asymptote indicates the AOA pointing from the midpoint of each sensor pair to the emitter. Thus, the AOAs are given by [19]

$$\theta_{a, k} = \tan^{-1} \left( \frac{y_2 - y_1}{x_2 - x_1} \right) \pm \cos^{-1} \left( \frac{r_k - r_1}{s_k - s_1} \right), \quad (8)$$

where the first term denotes the rotation angle of the TDOA hyperbola and the second term is the gradient angle of the hyperbola asymptote from the baseline between each sensor. The hyperbola pass through the midpoint of each sensor pair, which is given by

$$m_k = (s_k + s_k) / 2 \quad (9)$$

When $N-1$ sensor pairs are combined from $N$ sensors, we can obtain a set of linear equations from the gradient angles of asymptotes and the midpoints of each sensor pair as follows

$$e_i = A_i x + b_i \quad (10)$$

where

$$A_i = \begin{bmatrix} a_{i,1}^T \\ a_{i,2}^T \\ \vdots \\ a_{i,N-1}^T \end{bmatrix}, \quad b_i = \begin{bmatrix} a_{i,1}^T m_1 \\ a_{i,2}^T m_2 \\ \vdots \\ a_{i,N-1}^T m_{N-1} \end{bmatrix}$$

On the left-hand side of (10), $e_i$ denotes the measurement errors and the $2 \times 1$ vector $a_i$ consists of $[\sin \theta_{a, k} \cos \theta_{a, k} \cos \theta_{a, k}]$. Then, the WLS solution of the emitter position, $\hat{x}_e$, is given by [25]

$$\hat{x}_e = (A_i^T W_i A_i)^{-1} A_i^T W_i b_i \quad (11)$$

where the weighting matrix $W_i$ is defined by the inverse of the covariance matrix of TDOA measurement error $C_i$. The estimate of emitter position obtained in the first step is used to linearise a set of FDOA equations in the second step. Although the proposed method solves the problem using the AOA approximation for the simplicity, the existing TDOA-based closed-form solutions can be utilised.

Since the FDOA equation is related to both the position and the velocity of an emitter, as given by (5), we can estimate both the velocity and the position using FDOA measurements. However, since these position and velocity parameters of the emitter are coupled with each other in the non-linear FDOA equation, it is a challenging task to find a closed-form solution for these parameters without constraints on sensor placement. To simplify the problem, we modify the FDOA (5) using unit vectors connecting the sensors and the emitter as follows

$$\tilde{f}_k = -\frac{f_0}{c} \begin{bmatrix} (\tilde{x}_e - \tilde{s}_k)^T \tilde{t}_k - (\tilde{x}_e - \tilde{s}_k)^T \tilde{t}_k \end{bmatrix}, \quad k = 2, \ldots, N \quad (12)$$

In the modified FDOA equation in (12), FDOA measurement can be linearly related to the emitter velocity when the unit vectors $\tilde{s}_k$ and $\tilde{t}_k$ are estimated using the known sensor position and the estimate of emitter position obtained from the first step. Therefore, in the second step, we can transform non-linear FDOA equations to a set of linear equations by employing the estimated unit vector given by

$$\hat{t}_k = \frac{(\hat{x}_e - \hat{s}_k)^T}{\sqrt{(\hat{x}_e - \hat{s}_k)^T (\hat{x}_e - \hat{s}_k)}} \frac{(\hat{x}_e - \hat{s}_k)}{\hat{r}_k}, \quad k = 2, \ldots, N \quad (13)$$

where $\hat{x}_e$ and $\hat{t}_k$ denote the estimate of emitter position and the radius between the emitter and the sensor $k$, respectively. In the presence of FDOA measurement error, replacing the unit vector of each sensor in the FDOA equation by (13) results in the following FDOA measurement error

$$\tilde{f}_k = -\frac{f_0}{c} \begin{bmatrix} (\tilde{x}_e - \tilde{s}_k)^T \tilde{t}_k - (\tilde{x}_e - \tilde{s}_k)^T \tilde{t}_k \end{bmatrix}, \quad k = 2, \ldots, N \quad (14)$$

$$e_{z,k} = a_{z,k}^T \hat{x}_e + b_{z,k} = (\hat{t}_k - \hat{t}_k)^T \hat{x}_e - (\hat{s}_k^T \hat{t}_k - \hat{s}_k^T \hat{t}_k) + c_{z,k}, \quad k = 2, \ldots, N \quad (15)$$

where $c_{z,k}$ is the observed range-rate difference $\sigma^2 f_0 / f_0$. When $N-1$ sensor pairs are available, the set of modified FDOA equations is represented by a vector form given by

$$e_2 = A_2 \hat{x}_e + b_2 \quad (16)$$

where

$$A_2 = \begin{bmatrix} (\hat{t}_2 - \hat{t}_2)^T \\ (\hat{t}_3 - \hat{t}_3)^T \\ \vdots \\ (\hat{t}_N - \hat{t}_N)^T \end{bmatrix}, \quad b_2 = \begin{bmatrix} \hat{s}_2^T \hat{t}_2 - \hat{s}_2^T \hat{t}_2 - c_{z,2} \\ \hat{s}_3^T \hat{t}_3 - \hat{s}_3^T \hat{t}_3 - c_{z,3} \\ \vdots \\ \hat{s}_N^T \hat{t}_N - \hat{s}_N^T \hat{t}_N - c_{z,N} \end{bmatrix}$$

The WLS solution of the emitter velocity $\hat{x}_v$ is [21]

$$\hat{x}_v = (A_2^T W_2 A_2)^{-1} A_2^T W_2 b_2 \quad (17)$$

where the weighting matrix $W_2$ is defined by the inverse of the covariance matrix of FDOA measurement error $C_v$. 

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**Fig. 2 Illustration of the geometric relationship between TDOA hyperbola and its asymptote**
4 Simulation results

To validate the proposed method, computer simulations were conducted, and the performance was compared with the CRLB. In addition, the estimates of the proposed method were provided as an initial guess for the Taylor-series expansion method, and then we examined the final performance of the combined estimator. In the following simulations, the unit for positions is metres (m), and that for the velocity is metres per second (m/s). To estimate the position and the velocity of a moving emitter in a 2D space, at least two independent sensor pairs are required, because two pairs of TDOA/FDOA measurements have to be obtained. Although more sensor pairs would improve the estimation accuracy, the number of sensors is generally limited to achieve acceptable performance with the fewest sensors possible to minimise the cost and complexity of the overall system. Therefore we utilised three sensors in the following computer simulations.

The CRLB is the lowest variance of any unbiased estimators, and it bounds the achievable optimum performance under a given observation model of the probabilistic relationship between an observed data set and unknown parameters. CRLB is defined by the inverse of Fisher information, which is a measure for quantifying the amount of information about the unknown parameter in the observed data [25]. If the observed data are not at all related to the unknown parameters to be estimated, we could not estimate the unknown parameter from the data set. Assuming the general Gaussian case for TDOA/FDOA measurements [25], the Fisher information matrix is given by

\[
[J_{\text{fi}}] = \mathbf{H}^T \mathbf{C}_u^{-1} \mathbf{H} = \left[ \frac{\partial \mathbf{u}(\mathbf{p})}{\partial p_j} \right]^T \mathbf{C}_u^{-1} \left[ \frac{\partial \mathbf{u}(\mathbf{p})}{\partial p_j} \right] \tag{18}
\]

where \(\mathbf{H}\) is a Jacobian matrix of the TDOA/FDOA with respect to emitter parameters. The number of columns is equal to the number of unknown parameters, including the emitter position/velocity, and that of rows is the total number of TDOA and FDOA measurements. The matrix \(\mathbf{C}_u\) denotes a covariance matrix of measurement errors. The vector \(\mathbf{p} = [\mathbf{x}_1^T, \mathbf{x}_2^T]^T\) is an unknown parameter vector to be estimated that consists of the emitter position and velocity and \(\mathbf{u} = [\mathbf{r}^T, \mathbf{f}^T]^T\) consists of the TDOA and FDOA equations.

The TDOA and FDOA measurement errors were zero-mean Gaussian, and their variances were \(\sigma^2_{\text{TDOA}}\) and \(\sigma^2_{\text{FDOA}}\), respectively. The covariance matrices of TDOA and FDOA measurement errors were \(\mathbf{C}_{\text{TDOA}}\) and \(\mathbf{C}_{\text{FDOA}}\), where \(\mathbf{R}\) is a matrix with unity in the diagonal elements and 0.5 in the off-diagonal elements. The FDOA measurement error \(\sigma_r\) was set equal to 0.01\(\sigma_r\) to match the measurement range of the position (m) and the velocity (m/s) parameters for the given simulation environment. The scaling parameter 0.01 was determined to achieve a root mean square error (RMSE) below 100 m/s in terms of the CRLB. In this situation, when the standard deviation of TDOA measurement errors is \(\sigma_r = 1/c\) (0 dB on the x-axis of Figs. 4 and 5), that of range-difference errors \(\sigma_r\) becomes 1 m, and thus that of the range-rate-difference errors \(\sigma_r\) is equal to 0.01 m/s. The three sensors were deployed on the 2D space, and their positions were \(s_1 = [-100, 0]^T\), \(s_2 = [0, 0]^T\) and \(s_3 = [100, 0]^T\) with velocities of \(\mathbf{v}_1 = [20, 0]^T\), \(\mathbf{v}_2 = [30, 0]^T\) and \(\mathbf{v}_3 = [35, 0]^T\), respectively. The emitter was located at \(x_e = [1000, 1000]^T\) with a velocity of \(\mathbf{v}_e = [25, 0]^T\). The location geometry of the sensors and the emitter is shown in Fig. 3a. Since the sensors are located on a straight line, the existing QCLS-based methods are not applicable in this situation, whereas the proposed method enables us to estimate the position and velocity of a moving emitter.

Fig. 4 compares the accuracy of position and velocity estimates between the proposed method and the Taylor-series method in terms of RMSE as the noise variance increases. The number of ensembles was 10 000, and three iterations were performed for the Taylor-series method. The solid lines denote the CRLBs of position and velocity estimation. The RMSEs of the proposed method are given by the dash-dotted lines, and the dashed lines are the RMSEs of the Taylor-series method. The estimation accuracy of the proposed method approaches the CRLB at measurement error below −5 dB (the range-difference error \(\sigma_r\) is 0.3 m and the range-rate difference error \(\sigma_r\) is 0.003 m/s). The accuracy deviates for the CRLB and the Taylor-series method when the measurement error is larger than −5 dB, as shown in Fig. 4. The low performance of velocity estimation is because of the estimation error of the unit vector between the sensor and the emitter in (13), which causes the noise components in the system equations. Therefore the RMSE of velocity estimation by the proposed method increases steeply around a moderate noise level.

In Fig. 4, the initial guess of the Taylor-series method was set randomly around the true solution by adding independent zero-mean Gaussian noise with 1% mean square error (MSE), but this assumption of the accurate initial guess is not generally allowed in a practical situation. Therefore an alternative way of generating an appropriate initial guess is required to build a TDOA/FDOA-only emitter location system without any a priori information for an emitter. In this sense, the position and velocity estimates from the proposed method can be employed for the initial guess of the Taylor-series method.

Fig. 5 shows the performances of the Taylor-series method with three different strategies for setting the initial guess. We first set the initial guess by the proposed method and compared its
performance with that of the Taylor-series method with the randomly selected initial guesses, which were set randomly around the true solution by adding independent zero-mean Gaussian noise. This noise generates 5 and 12% MSE for the emitter position and velocity, respectively. In the case of the proposed method, the MSEs of the position and velocity estimates increase gradually, whereas those of the other cases are consistent at 5 and 12% MSE, as shown in the third subplot of Fig. 5. In particular, the MSE of velocity estimation is larger than that of position estimation, and the RMSE of velocity estimation starts to degrade severely around the measurement error of $-5$ dB. However, since finding a closed-form solution that produces a constant MSE is impractical, the proposed method can provide a coarse estimate for an initial guess of the emitter position and velocity without constraints on sensor placement.

To compare the proposed method with the existing QCLS method, we conducted the additional simulation with the four sensors deployed on the 2D space. Since the QCLS method cannot be applicable in case of the sensor deployment in a straight, the sensors were deployed without showing a common baseline, as shown in Fig. 3b. The positions of each sensor were $s_1 = [-100, 100]^T$, $s_2 = [0, 0]^T$, $s_3 = [100, 100]^T$ and $s_4 = [200, 0]^T$, with velocities of $\dot{s}_1 = [20, 0]^T$, $\dot{s}_2 = [30, 0]^T$, $\dot{s}_3 = [35, 0]^T$ and $\dot{s}_4 = [40, 0]^T$, respectively. Fig. 6 compared the accuracy of the

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**Fig. 4** Comparison of RMSE of the proposed method with the Taylor-series method and CRLB

Results are presented in log scale when the measurement error increases. Solid, dashed and dashed-dot lines show the CRLB, the RMSE of the Taylor-series method and the RMSE of the proposed method, respectively.

**Fig. 5** Comparison of RMSE of the Taylor-series method for three different initial guesses: the dotted and dashed-dot lines show the RMSE of the Taylor-series method with the randomly selected initial guesses 12 and 5% in the first and second subplots, respectively. Dashed lines are the RMSE of the Taylor-series method initiated by the estimates from the proposed method. Third subplot shows the MSE (%) of the proposed method as the measurement error increases.
proposed method with that of the QCLS method. The proposed method yielded the comparable performance to the QCLS method at the low measurement error under $-5$ dB, but deviated from the CRLB when the measurement error became larger than $-5$ dB in both cases of the position and velocity estimation. The performance degradation of position estimation came from the discrepancy between the approximated hyperbolic asymptotes and the exact TDOA curves in the first step. Since the FDOA equations were linearised using the estimated position in the first step, the performance degradation was relatively larger in the velocity estimation as shown in Fig. 6b. Alternatively, the proposed method can be linked to the existing iterative algorithms to improve the estimation performance. In conclusion, although the proposed method achieves a relative lower estimation accuracy, it can be applicable to any sensor deployment scenarios while the QCLS method does not work at all for the sensor geometry of a straight line because of its singularity problem in a system matrix [11].

5 Conclusions

A closed-form solution for estimating the position and the velocity of a moving emitter was developed for TDOA/FDOA-based emitter localisation using a two-step approach in a 2D scenario. In the first step, the position of a moving emitter is derived by estimating the intersection of asymptotes of TDOA hyperbolas rather than the quadratic TDOA hyperbolas themselves. The FDOA equations are transformed to a set of linear equations by replacing the unit vectors for each sensor, and then the velocity of the moving emitter is estimated based on the WLS method. Although the method achieved lower accuracy than other emitter location methods, the method provides a closed-form solution without any constraints of sensor geometry.

Many methods resort to a numerical iterative algorithm, and closed-form methods are constrained by sensor placement, but the proposed method is an appropriate alternative to overcome these problems. Particularly, in methods based on a numerical iterative approach, the proposed two-step method can provide a proper initial guess without a priori information. Hence, we expect that the method would be helpful for implementing a TDOA/FDOA-based localisation system for a moving emitter by combining established algorithms. In terms of the sensor placement, the proposed method can be applicable without any geometric constraints for 2D scenarios. However, for the practical usefulness, the proposed method needs to be extended to a 3D scenario in future works.

6 Acknowledgements

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (2014R1A1A2056013).

7 References


Fig. 6 Comparison of RMSE between the QCLS method and the proposed method
Dashed lines are the RMSE of the QCLS method and dashed-dotted lines are the RMSE of the proposed method.