Unbiased Equation-Error Adaptive IIR Filtering Based on Monic Normalization

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***Abstract*— We present a novel way to remove the bias in equation-error based adaptive infinite impulse response (IIR) filtering by conceiving a scheme called *monic normalization*. It is found that normalizing all the coefficients of the denominator filter by the first coefficient after each adaptation removes the bias and leads to unbiased estimates. The analysis of stationary points is presented to show that the proposed method can indeed produce unbiased parameter estimates in the presence of noise. The computer simulation results also demonstrate that the proposed method performs better than or comparable to existing algorithms, while requiring much lower computational complexity.**

***Index Terms*—Adaptive IIR filtering, equation error, unbiased estimate.**

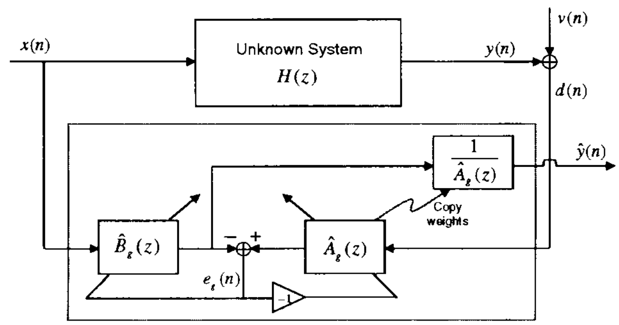

Fig. 1. Proposed adaptive IIR filter in the system identification configuration.

I. Introduction

Equation-error adaptive infinite impulse response (IIR) filtering has some attractive features such as a unimodal error surface, good convergence, and guaranteed system stability, compared to output-error adaptive IIR filtering [1]. In spite of those advantages, the equation-error approach has not been widely used, since it may generate biased coefficient estimates in the presence of noise.

Most existing algorithms in equation-error adaptive IIR filtering have tried to remove the bias through additional filtering or cumbersome noise suppression procedures [2]–[4]. Recently, the unit-norm constraint has drawn much interest as an appropriate alternative for solving the bias problem without resorting to filtering or noise suppression [5], [6]. The focus of development of such algorithms is to make the process of coefficients update unaffected by noise.

In this letter, we introduce a novel scheme called *monic normalization* for unbiased equation-error adaptive IIR filtering. The proposed method is inspired by the monic constraint commonly used in the existing equation-error algorithms, in which the first coefficient of the denominator filter is fixed to unity. In adaptive IIR filtering with monic normalization, all the denominator coefficients of an adaptive IIR filter including the first coefficient are adapted and normalized by the updated first coefficient after each iteration. In this way, the noise effect on the updated coefficients can be removed because the noise effect is shown to be approximately identical for all those coefficients.

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II. Adaptive IIR Filtering with Monic Normalization

The proposed equation-error adaptive IIR filter applied in the system identification configuration is shown in Fig. 1. Let the zero-mean discrete-time signal $x(n)$ be an input signal to an unknown system H. If the unknown system H is modeled by a rational system function which consists of the numerator filter coefficients b_k, $0 \le k \le K$ and the denominator filter coefficients a_l, $0 \le l \le L$, then the output $y(n)$ of the unknown system H is represented by a difference equation such that

$$y(n) = \frac{1}{a_0}\left(\sum_{k=0}^{K} b_k x(n-k) - \sum_{l=1}^{L} a_l y(n-l)\right) \quad (1)$$

where without loss of generality it is assumed that $a_0 = 1$. When the output of H is corrupted by additive noise, the observed output of H, or the desired signal is represented by

$$d(n) = y(n) + v(n) \quad (2)$$

where $v(n)$ is zero-mean white measurement noise with variance σ_v^2. The problem is to estimate a_k and b_l given $x(n)$ and $d(n)$.

If the sufficient order modeling situation is considered here, the functions $\hat{A}_g(z)$ and $\hat{B}_g(z)$ are given by

$$\hat{A}_g(z) = \sum_{l=0}^{L} \hat{a}_l z^{-l} \quad \text{and} \quad \hat{B}_g(z) = \sum_{k=0}^{K} \hat{b}_k z^{-k} \quad (3)$$

where $\hat{a}_k$ and $\hat{b}_l$ are the adaptive filter coefficients estimating a_k and b_l, respectively, and z^{-k} denotes a delay of k samples. Note that this modeling is different from the conventional equation-error approach in which the value corresponding to $\hat{a}_0(n)$ is fixed to unity. By using the adjustable first coefficient $\hat{a}_0(n)$, we try to measure the effect of noise on the updated coefficients. It will be shown in Section III that the noise effect on the updated coefficients is approximately identical for all those denominator filter coefficients. Since the first coefficient of the unknown system, a_0, is the *a priori* known value of unity, measuring of the noise effect can be roughly achieved by the updated first coefficient. So, we can suppress the noise effect by the *monic normalization* procedure in which all the coefficients of the denominator filter are normalized by the first coefficient $\hat{a}_0(n)$, so that the denominator polynomial $\hat{A}_g(z)$ becomes a monic polynomial.

First, we define a *generalized equation error* involving the adjustable coefficient $\hat{a}_0(n)$ by

$$e_g(n) = \hat{\boldsymbol{a}}^T(n)\boldsymbol{d}(n) - \hat{\boldsymbol{b}}^T(n)\boldsymbol{x}(n) \tag{4}$$

where $\hat{\boldsymbol{a}}^T(n) = [\hat{a}_0(n), \hat{a}_1(n), \cdots, \hat{a}_L(n)]$, $\hat{\boldsymbol{b}}^T(n) = [\hat{b}_0(n), \hat{b}_1(n), \cdots, \hat{b}_K(n)]$, $\boldsymbol{d}^T(n) = [d(n), d(n-1), \cdots, d(n-L)]$, and $\boldsymbol{x}^T(n) = [x(n), x(n-1), \cdots, x(n-K)]$. It should be noted that when $\hat{a}_0(n) = 1$, the generalized equation error reduces to the conventional equation error $e_e(n)$ in [1]. By minimizing the mean-square error (MSE), $E[e_g^2(n)]$, we find the solutions of $\hat{\boldsymbol{a}}(n)$ and $\hat{\boldsymbol{b}}(n)$. Generally, to avoid the trivial solution $\hat{\boldsymbol{a}}(n) = \hat{\boldsymbol{b}}(n) = \mathbf{0}$, the "monic" constraint, $\hat{a}_0(n) = 1$, or the "unit-norm" constraint, $||\hat{\boldsymbol{a}}(n)|| = 1$, has been placed on the adjustable coefficient vector [5]. Although the monic constraint is maintained in the proposed method, $\hat{a}_0(n)$ is adapted unlike conventional equation-error algorithms.

The stochastic gradient method minimizing $E[e_g^2(n)]$ produces the following update equations:

$$\hat{\boldsymbol{b}}(n+1) = \hat{\boldsymbol{b}}(n) + \mu_b e_g(n)\boldsymbol{x}(n) \tag{5}$$

$$\tilde{\boldsymbol{a}}(n+1) = \hat{\boldsymbol{a}}(n) - \mu_a e_g(n)\boldsymbol{d}(n) \tag{6}$$

$$\hat{\boldsymbol{a}}(n+1) = \frac{1}{\tilde{a}_0(n+1)}\,\tilde{\boldsymbol{a}}(n+1) \tag{7}$$

where μ_b and μ_a are the step sizes that control the convergence speed and the residual error after convergence. The monic normalization step of (7) is introduced in order to ensure that the noise effect is suppressed after each iteration and makes it possible to overcome the bias problem. After each update iteration, the denominator filter coefficient, $\hat{\boldsymbol{a}}(n)$, is copied to the all-pole part of the IIR filter (Fig. 1).

III. Analysis of Stationary Points

Let $e_s(n)$ be a noise-free error defined by

$$e_s(n) = \hat{\boldsymbol{a}}^T(n)\boldsymbol{y}(n) - \hat{\boldsymbol{b}}^T(n)\boldsymbol{x}(n) \tag{8}$$

where $\boldsymbol{y}(n) = [y(n), y(n-1), \cdots, y(n-L)]^T$. Then we can rewrite the update equation (6) as

$$\begin{aligned}\tilde{\boldsymbol{a}}(n+1) = {} & \hat{\boldsymbol{a}}(n) - \mu_a e_s(n)\boldsymbol{y}(n) - \mu_a[\boldsymbol{v}^T(n)\hat{\boldsymbol{a}}(n)]\boldsymbol{v}(n) \\ & - \mu_a e_s(n)\boldsymbol{v}(n) - \mu_a[\boldsymbol{v}^T(n)\hat{\boldsymbol{a}}(n)]\boldsymbol{y}(n)\end{aligned} \tag{9}$$

where $\boldsymbol{v}^T(n) = [v(n), v(n-1), \cdots, v(n-L)]$ is the noise vector. We assume that the filter inputs and the adapting coefficients are independent [7]. Correspondingly, the coefficient vector $\hat{\boldsymbol{a}}(n)$ is independent of $\boldsymbol{v}(n)$. When expectation is taken and the independency assumptions are applied, (9) now becomes

$$\begin{aligned}E\{\tilde{\boldsymbol{a}}(n+1)\} = {} & E\{\hat{\boldsymbol{a}}(n)\} - \mu_a E\{e_s(n)\boldsymbol{y}(n)\} \\ & - \mu_a E\{[\boldsymbol{v}^T(n)\hat{\boldsymbol{a}}(n)]\boldsymbol{v}(n)\}.\end{aligned} \tag{10}$$

The last term on the right-hand side of (10) causes the bias problem in the conventional equation-error adaptive IIR filtering. In the proposed method, however, the normalization process of (7) plays a key role in solving the bias problem. Since $v(n)$ is assumed to be white, using $E\{[\boldsymbol{v}^T(n)\hat{\boldsymbol{a}}(n)]\boldsymbol{v}(n)\} = \sigma_v^2 E\{\hat{\boldsymbol{a}}(n)\}$ in (10) produces

$$E\{\tilde{\boldsymbol{a}}(n+1)\} = (1-\mu_a\sigma_v^2)E\{\hat{\boldsymbol{a}}(n)\} - \mu_a E\{e_s(n)\boldsymbol{y}(n)\}. \tag{11}$$

From (11), we observe that the noise effect in adapting the coefficients is identical for all the coefficients. The value of this noise effect can be approximately measured from the updated first coefficient. The first element of $\hat{\boldsymbol{a}}(n)$, $\hat{a}_0(n)$, becomes one after the normalization process of the nth iteration. By using $E\{\hat{a}_0(n)\} = 1$ and $(1-\mu_a\sigma_v^2) \gg \mu_a E\{e_s(n)y(n)\}$, we obtain

$$E\{\tilde{a}_0(n+1)\} \approx 1 - \mu_a\sigma_v^2. \tag{12}$$

For small values of μ_a, we can show that $\tilde{a}_0(n+1)$ is independent of $\tilde{a}_i(n+1)$, $i = 1, 2, \cdots, L$. Hence, normalizing both sides of (11) by $E\{\tilde{a}_0(n+1)\}$ yields

$$E\{\hat{\boldsymbol{a}}(n+1)\} = E\{\hat{\boldsymbol{a}}(n)\} - \frac{\mu_a}{1-\mu_a\sigma_v^2}\,E\{e_s(n)\boldsymbol{y}(n)\}. \tag{13}$$

If a stationary point is reached, i.e., $E\{\hat{\boldsymbol{a}}(n+1)\} = E\{\hat{\boldsymbol{a}}(n)\}$, (11) reduces to

$$E\{e_s(n)\boldsymbol{y}(n)\} = \mathbf{0} \tag{14}$$

where $\mathbf{0}$ is a zero vector of appropriate size. The result in (14) is different from the corresponding result of the conventional equation-error case, $E\{e_s(n)\boldsymbol{y}(n)\} = -\sigma_v^2 E\{\hat{\boldsymbol{a}}(n)\}$ as can be seen in (11). This implies that no bias exists at the stationary point in the proposed adaptive IIR filtering method. The noise component results in the minute increase of the step size. However, it does not alter the convergence behavior because $\mu_a\sigma_v^2$ is usually smaller than 10^{-3} in most cases.

IV. Simulation Results

Computer simulations are carried out in the system identification configuration to illustrate the good convergence property of the proposed algorithm. Both the EE algorithm, which denotes the conventional equation-error algorithm, and the QCEE algorithm [6] are compared with the proposed algorithm. The EE algorithm is chosen for comparison of bias-removal capability and the QCEE algorithm is for comparison of convergence speed and estimation accuracy. The unknown system, $H(z)$, was driven by a white, zero-mean, Gaussian random sequence having unit variance. The measurement noise sequence is white Gaussian with variance σ_v^2. The squared norm of the parameter estimation error, $||\boldsymbol{a} - \hat{\boldsymbol{a}}(n)||^2 + ||\boldsymbol{b} - \hat{\boldsymbol{b}}(n)||^2$, is taken and averaged over 20 independent trials. The signal-to-noise ratio (SNR) is calculated by SNR $= 10\log(E[y^2(n)]/E[v^2(n)])$.

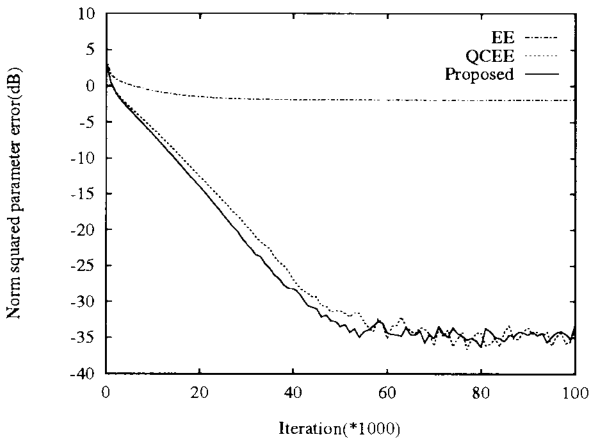

Fig. 2. Plot of the norm squared parameter errors of Example 1.

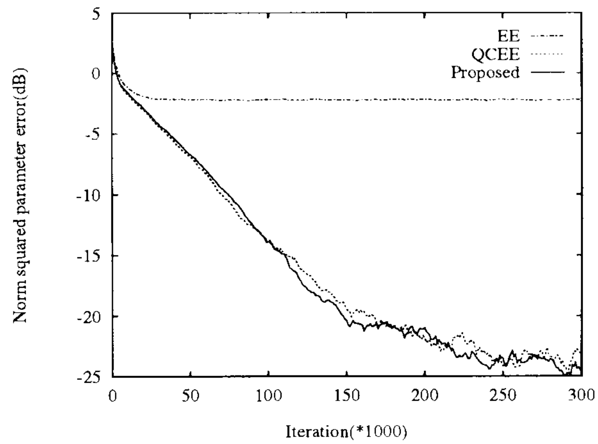

Fig. 3. Plot of the norm squared parameter errors of Example 2.

Example 1: The transfer function of the unknown system is given by

$$H(z) = \alpha \frac{1 - 0.5z^{-1}}{1 - 1.0z^{-1} + 0.5z^{-2}} \tag{15}$$

which has two poles at $0.5\angle \pm 45^\circ$ and one zero at 0.5. The gain α is chosen such that $y(n)$ has unit power, and is similarly chosen for other simulations. For this example, we have $\alpha = 0.8465$ and the SNR is 10 dB. All the algorithms use the same step size of 0.001. Fig. 2 shows the learning curves of the norm squared parameter errors. The proposed algorithm has the good performance similar to the QCEE algorithm, while the EE algorithm converges to a biased solution and shows poor estimation accuracy due to the presence of noise. Compared with the EE algorithm, the proposed algorithm has about 30 dB better performance.

Example 2: The transfer function of the unknown system is given by

$$H(z) = \alpha \frac{(1 - 0.9z^{-1})(1 + 0.81z^{-2})}{(1 - 1.34z^{-1} + 0.90z^{-2})(1 + 0.75z^{-1} + 0.56z^{-2})} \tag{16}$$

which has four poles at $0.95\angle \pm 45^\circ$, $0.75\angle \pm 120^\circ$, and three zeros at $0.9\angle \pm 90^\circ$, 0.9. For this example, we have $\alpha = 0.5361$ and the SNR is 0 dB. All the algorithms use the same step size of 0.0002. Fig. 3 shows the learning curves of the norm squared parameter errors. Although the order of the unknown system is increased and the SNR is decreased, the proposed algorithm still shows the good convergence property and has a comparable performance with the QCEE algorithm in terms of the convergence speed and the estimation accuracy. However, the EE algorithm converges to a biased solution and its norm-squared error is about 20 dB larger than that of the proposed algorithm.

While the performances of the proposed algorithm are almost the same as those of the QCEE algorithm in terms of the convergence speed and the estimation accuracy, the computation time required for each iteration of the proposed algorithm is much less than that of the QCEE algorithm. This is because the proposed algorithm does not contain any matrix multiplication and computation of the norm of the coefficient vector required by the QCEE algorithm [6].

V. Conclusions

We have presented an efficient bias removal algorithm based on monic normalization for equation-error adaptive IIR filtering. The proposed algorithm does not contain any additional filterings or matrix multiplication required in the existing bias removal algorithms, which enables the proposed algorithm to produce unbiased estimates with much lower complexity. In addition, the convergence performance of the proposed algorithm has been shown to be better than or comparable to the existing algorithms.

Although the undermodeled case and the colored measurement noise are not considered in this letter, the results in [5] and [6] can be readily applied to the proposed algorithm and much of the related issues are left for future work.

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